Agitation of Non-Newtonian Fluids

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Since the shear rate of a non-Newtonian fluid is of importance in fixing the rheological or viscometric behavior of such a material, the present study has been concerned with the development of a general relationship between impeller speed and the shear rate of the fluid. The resulting relationship was then used to interpret and correlate power-consumption data on three non-Newtonian fluids by use of a generalized form of the conventional power-number–Reynolds-number plot for Newtonians.

Flat-bladed turbines from 2 to 8 in. in diameter were used exclusively. Tank diameters ranged from 6 to 22 in. and power inputs from 0.5 to 176 hp./1,000 gal. The study encompassed a 130-fold range of Reynolds numbers in the laminar and transition regions. The results to date indicate that power requirements for the rapid mixing of non-Newtonian fluids are much greater than for comparable Newtonian materials.

The literature in the field of agitation and mixing of fluids may be divided into two general categories. First and most extensive are the numerous papers dealing with the dependence of power consumption upon geometric, kinematic, and fluid-property parameters. The broad fundamental and applications studies of Mack, Miller, Oldshue, Rushton, and various coworkers (*5, 8, 15, 21, 22,* for example) are representative of work of this nature. Studies of rates and quality of mixing, and of the effect of agitator geometric variables on these factors comprise the second group. The complexity of this problem has resulted in little quantitative progress in this field except in the mixing of particulate solids, and the literature, therefore, presents primarily qualitative guides as opposed to the quantitative generalizations available for prediction of power requirements. The most recent review by Rushton (*20*) discusses progress in this field.

A major limitation of the prior art is that it deals almost exclusively with Newtonian fluids. The comparative simplicity of Newtonian systems has made them the logical first choice for exploratory work, but because of the importance of non-Newtonian materials a study of these systems is long overdue. The purposes of the present investigation therefore were as follows: (1) establishment of the quantitative relationships between power consumption and the geometric, kinematic, and fluid property parameters for non-Newtonian fluids; (2) qualitative study of rates and quality of mixing in non-Newtonian systems to shed some preliminary light on these factors; and attaining (3) a quantitative method of approach in classification of non-Newtonian fluids and ascertaining its relation to mixing.

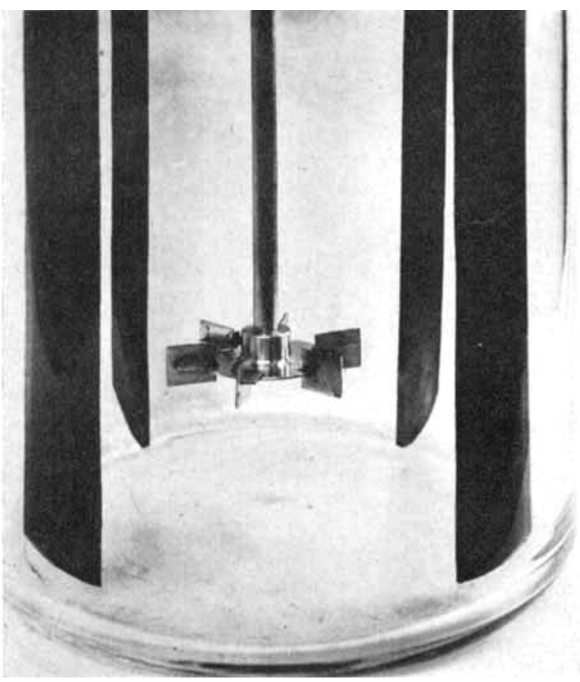

Fig. 1. Mixing system.

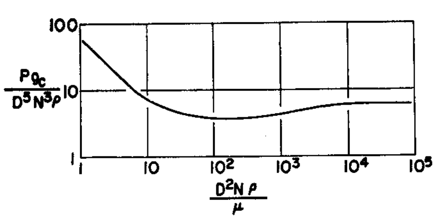

Fig. 2. Power-number–Reynolds-number curve for Newtonian fluids in a baffled tank (*22*).

REVIEW OF PRIOR ART

In the study of the agitation or mixing of fluids, the system which has received the most attention consists of a single impeller centered in a cylindrical tank, as shown in Figure 1. The results of Newtonian power-consumption studies are presented in terms of dimensionless groups involving power ($Pg_c/D^5N^3\rho$) and a mixing Reynolds number ($D^2N\rho/\mu$) or modifications of these. Below a Reynolds number of 300 the Froude number (DN^2/g), which measures the variation in flow due to changes in the free surface, was not important (*22*). The effects of geometrical parameters other than the impeller diameter [such as (C/D), (T/D), and (B/D)] were not important within the wide ranges specified by previous workers. The entire power-number–Reynolds-number curve (Figure 2) has been divided into three sections which are directly analogous to the familiar three regions of flow in a circular pipe, i.e., the turbulent region in which the power number (or friction factor) is not greatly affected by Reynolds number, the laminar region where power number and friction factor are inversely proportional to Reynolds number, and the intermediate or transition region. However, unlike the case of flow in a round tube, the transition from laminar to turbulent flow does not occur over the narrow range of Reynolds numbers between 2,000 and 4,000 (*6*) but extends over the large range from 10 to 1,000, as shown in Figure 2.

To date only three papers have been concerned with the agitation-power requirements of non-Newtonian fluids. Brown and Petsiavas (*2*) presented a power-number plot for a Bingham-plastic type* of non-Newtonian that makes use of the Bingham-plastic Reynolds and Hedstrom numbers (*4, 13*) to correlate the data. Their viscometric data, taken with a Brookfield viscometer, indicated that their fluids deviated appreciably from Bingham-plastic behavior, but for those few fluids which closely approach the ideal Bingham plastic this method of attack can be reworked into

*Appendix A presents and discusses the classical types of non-Newtonian behavior.

a rigorous and convenient design method. Magnusson (*9*) reported a procedure for calculating the apparent viscosity of non-Newtonian fluids in agitated tanks by comparison with the power number curve for a Newtonian fluid but presented no method whereby such results might be used for equipment design. Schultz-Grunow (*23*) studied power requirements for the agitation of slightly non-Newtonian fluids in the laminar region and developed a correlation by means of dimensional analysis. Since fluid density was not included in the analysis, this correlation cannot be used outside the laminar region and may not accurately predict the end of the laminar-flow region. These are rather serious limitations in view of the fact that little actual mixing of non-Newtonian fluids appears to occur within the laminar region.

Summarizing, it may be concluded that the prior art presents indications of several different approaches to the problem of non-Newtonian power consumption in agitated vessels. However, no useful equipment-design methods have been developed and no over-all physical understanding of the problem has as yet been presented.

DESCRIPTION AND CLASSIFICATION OF NON-NEWTONIAN FLUIDS

In view of the previously reported (*12, 13*) limitations of the common rheological classifications of non-Newtonian behavior, the method of approach suggested in this work is to define the extent of non-Newtonian behavior by the property n, as given by the power-law equation:

$$\tau = K\left(\frac{du}{dr}\right)^n \tag{1}$$

Newtonian fluids are defined as those materials which in laminar flow exhibit a linear relationship between the imposed shear stress and the resulting shear rate; i.e., n has a constant value of unity and K is equal to μ/g_c where μ is termed the viscosity of the Newtonian fluid. Since the property n is a measure of the type of fluid behavior, it has been termed the *flow-behavior index* of a fluid (*13*). Similarly, K may be considered to be a *fluid-consistency index*.

Non-Newtonian fluids are those materials for which the flow-behavior index is not equal to unity although it is frequently a constant, or nearly so, over wide ranges of shear rates. The fact that is not a true constant—over all conceivable ranges of shear rate—is frequently irrelevant in view of the fact that one needs a rheological equation which correctly portrays only the fluid behavior over the particular range of shear rates which happen to be of interest in a given engineering problem. The rigor and utility with which a similar method of approach has been applied to the analogous problem of flow in pipes (*12*) suggest its utility in the field of agitation. As a matter of fact, for flow in pipes the property analogous to n in Equation (1) does not need to be a constant in order for the analysis to be rigorous, but may be allowed to vary with the shear rate of the fluid. The problem of agitation of fluids in vessels has been too complex to enable a similar theoretical analysis to date, and one cannot yet say whether or not the exponent n must be a constant to make the approach a rigorous one. From an engineering viewpoint it is important to note that no fluids have yet been found for which n changes rapidly enough with shear rate to permit an experimental analysis of this problem or to reveal any discrepancies of data attributable to this factor. Severs (*25*) has suggested that studies using dilatant fluids, for which n sometimes does change rapidly with shear rate, would be illuminating.

In view of the mathematical simplicity of Equation (1) it is remarkable that it correlates rheological data as well as it does. For example, the data of reference *3* are correlated at least as perfectly by Equation (1) as by the much more complex Eyring-Powell equation used by Stevens et al. This statement of fact may be readily verified by simply plotting both the rheological data and the Eyring-Powell equation for each fluid (as given by reference *3*) on logarithmic coordinates and comparing the fit obtained with that obtainable by the simple straight line which depicts Equation (1) on such coordinates. The rheological data of the fluids used in the present work (Figure 5) do not fall on a perfect straight line over the entire range of shear rates for which data are available, but for all fluids except the 1.0% CMC the deviation from such a straight line is less than the scattering of the data points. Even for this CMC gel the data which are within the range of average shear rates used in the agitation studies fall on a perfect straight line.

DEVELOPMENT OF THE CORRELATION

The flow of a fluid about a mixing impeller is complex, but the controlling factors of the drag on an impeller blade may be examined by comparison with simpler flow situations. Figure 3 shows the laminar flow of a fluid around a sphere. There is no separation of the flow and viscous dissipation of energy is the controlling factor. The total force on the sphere is due to two effects: (1) the shearing forces on the surface of the object (of magnitudes indicated approximately by arrows tangent to the surface on Figure 3) and (2) the differences in pressure between the front and the rear of the sphere. These pressure differences in turn are due to the viscous dissipation of energy in the fluid stream and are indicated by the arrows perpendicular to the surface of sphere.

For a Newtonian fluid two thirds of the drag is due to shear-stress forces on the surface (*17*), the remainder being due to differences in pressure caused by the flow of material in the region of the surface. For a flat plate of zero thickness normal to the flow of fluid (Figure 4), which may be assumed similar to a flat-bladed turbine or paddle rotating in a fluid, *all* the drag is due to differences in pressure caused by the flow of the fluid in the region of the object, as there is no area along which the shear stress at the surface can exert a component causing a net drag in the direction of the motion of the fluid.* One may conclude from this analogy that the study of such an object moving through the fluid must include the study of the fluid in the general region of the object, since this determines the viscous-energy dissipation and hence the drag or power requirement.

In non-Newtonian technology, study of the flow in this over-all region must obviously include a consideration of the shearing rates or shearing stresses. This is a necessary consequence of the fact that the response (viscosity) of these fluids to an imposed stress is not a constant but depends on the magnitude of either the shearing rate or shearing stress. Another way of arriving at the same conclusion is to note that no simple equation for the relationship between shear stress and shear rate [such as Equation (1)] has been found which will correlate such data over all conceivable ranges of these variables. Therefore, it is necessary to know at least approximately the ranges into *which the shear rates about an impeller may fall and the variables which determine or control these shear rates*. Figure 5 shows the shear-stress–shear-rate relationships for the materials used. Evaluation of the ratio of shear stress to shear rate (apparent viscosity) at a given point showed for the 1.25% Carbopol solution a variation in apparent viscosity from 134,000 centipoises at a shear rate of 1.5 sec.$^{-1}$ to 1,100 centipoises at 1,000 sec.$^{-1}$. With such a large variation in apparent viscosity one could not make a one-point measurement of the apparent viscosity indiscriminately and expect to obtain a correlation when the range of shear rates in the viscometer was different from that of the shear rates in the system being studied.

The mathematical description of a non-Newtonian material may be accomplished by the use of an equation [such as Equation (1)] relating the shear stress to shear rate or, equally well, by the

*This analogy of flow over a flat plate as compared with flow past a mixing impeller is not completely exact as the postulated types of flow do not include the radial components of velocity which are parallel to the surface of the flat plate or impeller. Such radial flow is necessary to produce mixing. This does not invalidate the argument, however, because the drag due to radial flow does not produce any forces perpendicular to the surface of the impeller and therefore does not directly affect the torque felt by the shaft and hence the power consumption of the system.

ratio of its shear stress to shear rate as a function of, say, shear rate. If one chooses the latter, one has a simple comparison with a Newtonian fluid because in the limiting case, a Newtonian, the ratio of shear stress to shear rate is not a function of shear rate. Of course, the choice is arbitrary. The latter was chosen in this case to simplify the visualization of the behavior of the non-Newtonian fluid.

In order to pinpoint the shear-rate range in a mixing system, one may devise a means of measuring the apparent viscosity of the system and then determine its relation to the other variables of the system. To define apparent viscosity one may consider two identical sets of mixing equipment, one of which contains a Newtonian fluid and the other a non-Newtonian. If these fluids are agitated in the laminar region, with the same impeller speed used in each, and one varies the viscosity of the Newtonian by diluting it or thickening it so that the power measured at each impeller is the same, then, because all variables are identical, one may say that the average viscosities are the same in both pieces of equipment. Upon measuring the viscosity of the Newtonian fluid one knows the apparent viscosity of the non-Newtonian existing under the given experimental conditions.

The preceding experiment defines the apparent viscosity in the system. It is necessary to determine its dependence on the variables of the system. In this paper it is assumed that the fluid motion in the general region of the impeller can always be characterized by an average shear rate which is linearly related to the rotational speed of the impeller, viz:

$$\left(\frac{du}{dr}\right)_{average} = kN \qquad (2)$$

If this line of reasoning is followed, the only necessary fluid properties are the apparent viscosity and density, although the latter would not be expected to be a significant variable in the laminar region. Until means are developed both for measuring and averaging shear rates the evaluation of k in Equation (2) must be done indirectly.

In Figure 6 the power number for a 2.5% CMC solution is plotted as a function of impeller speed in the system previously described. Since the comparable Newtonian data for this system are known (*22*), the viscosity which a Newtonian fluid would have under identical conditions of speed and power consumption in the same equipment can be obtained merely by referring to the Newtonian power-number curve. According to the preceding arguments, this must be identical to the apparent viscosity of the non-Newtonian. When the apparent viscosity is thus obtained, reference may be made to the viscometric curve for the flui (such as Figure 5) to obtain the corresponding average shear rate in the system. The best value for the proportionality constant k in Equation (2), obtained in this manner, was 13. Consideration was given to all fluids and T/D ratios studied in the laminar region. It may be noted that only data in the laminar region are useful for this purpose as power data outside the laminar region are insensitive to viscosity.

RESULTS

The non-Newtonian materials tested were two colloidal suspensions, CMC or sodium carboxymethylcellulose (Hercules Powder Company) and Carbopol 934 (Goodrich Chemical Company) and a suspension of Attasol clay (Attapulgus Mineral and Chemical C npany). As shown in Figure 5, two con ntrations of CMC and of Carbopol nd one of Attasol were used. On a logarithmic diagram such as Figure the linear shear-stress–shear-rate rel ionship of Newtonian fluids appears a straight 45-deg. line (n = 1.00). T e deviation from Newtonian behavior, therefore, is indicated by the divergenc of the slope from the value of unity. Attasol was accordingly the most n -Newtonian (n = 0.24) of the fluids te ed. Because all these materials were o the pseudoplastic type the slopes of th curves were all less than unity. The Attasol-clay suspensions of the concentration studied are frequently termed *Bingham plastic* by the engineering literature and the other materials are commonly, and correctly, termed *pseudoplastic.*

The apparent viscosity of a given material at a particular shear rate was determined by dividing the shear stress by the particular shear rate. An important point to note here is that the shear rates in Figure 5 were not calculated by presuming a shear-stress–shear-rate relationship, nor were they calculated by assuming that the shear rates were those which a Newtonian fluid would exhibit in the same viscometer. Both these erroneous attacks, common in the literature, are not necessary at the present state of the art. (Cf. Appendix C.) The variation of the viscometric data with temperature was shown to be negligible for the small variations in room temperature encountered and were not always recorded.

Figure 7 shows the power correlation for all the materials tested. Of over 130 data points taken, all but ten lie within 15% of the curve drawn through the data. The ranges of data were as follow:

Impeller diameters	2 to 8 in.
Tank diameters	6 to 22 in.
T/D	1.3 to 3.7 (laminar region)
	2.0 to 5.5 (transition region)
Power input	0.5 to 176 hp./1,000 gal.
Speed	95 to 1,190 rev./min.
Apparent viscosity	7 to 180 poises
Reynolds number	2.0 to 270

Both baffled ($B = 0.1T$) and unbaffled systems were studied. The equipment used is described in Appendix B; the

TABLE 4. MIXING-RATE DATA

Run	Fluid	Minimum h.p./1,000 gal.*	Minimum N_{Re}*	T/D
Movement at wall of tank				
2	2.0% CMC	40.2	87.4	2.95
14	Attasol	21.6	122.	2.95
3A	1.0% CMC	8.66	94.0	2.95
12	Carbopol	50.0	49.0	1.97
†	Carbopol	74.0	42.5	2.00
15	Attasol	16.8	75.8	1.97
17	Attasol	5.09	32.4	1.48
13	Carbopol	27.7	9.8	1.33
Movement of fluid surface				
3A	1.0% CMC	53.1	268.	2.95
8	1.0% CMC	17.3	81.7	3.00
15	Attasol	35.8	113.	1.97
5	1.0% CMC	23.0	94.0	2.00
17	Attasol	35.0	98.0	1.48
	Median of all:	25–35	50–100	

*Minimum values at which fluid movement could first be observed visually.
†Run taken in conjunction with run 10. *See* footnote ** in Table 3 [footnote on page 6].

detailed dimensions of the equipment and the experimental data are tabulated in Tables 1 to 4.*

DISCUSSION OF RESULTS

Laminar Region

With the exception of 1.0% CMC and the Attasol suspension, Table 3 and Figure 7 show that all fluids were studied in the laminar region in at least one run. The agreement between the data and the conventional Newtonian correlating curve is good.

Transition Region

In general, the gradual development of turbulence and departure from laminar behavior may be stated to be due to the formation of eddies. Consideration of the flow properties of the pseudoplastic fluids used in this work shows that their apparent viscosity must increase with increasing distance from the impeller, in view of the lower velocities, hence shear rates, of the fluids as they leave the vicinity of the impeller. A small volume of fluid leaving the high-shear-rate region near the impeller therefore encounters progressively more viscous material. This, in turn, would cause a depressive effect on the propagation of eddies. The last factor would tend to decrease the rapidity with which the power-number–Reynolds-number curve begins to diverge from the 45-deg. line of the laminar region as the fluid becomes more non-Newtonian in the direction of greater pseudoplasticity (n approaching zero). The net result of this behavior should be an extension of the laminar region to Reynolds numbers above 10 or to power numbers below 7.1, the point at which Newtonian fluids enter the transition region. Figure 7 shows that such a retardation was indeed observed. It must be emphasized that this extension of the laminar region is not due to the particular form of the Reynolds number chosen in this work. The power number is used in its conventional orm, and since Newtonian systems leave the laminar region at a power number of 7.1 no conceivable change in Reynolds number could shift the curve to coincide with the usual Newtonian behavior, shown as a dashed line in Figure 7. Furthermore, the data indicate that baffling had no effect on power consumption under the conditions evaluated; hence this extension of the laminar region was found to be common to baffled as well as to unbaffled systems. The more gradual transition from laminar to turbulent flow for pseudoplastic fluids has also been shown to occur in flow through pipes (*12*), and Brown and Petsiavas (*2*) noted the same behavior in independent mixing studies.

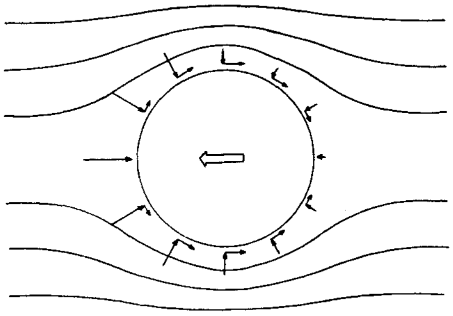

Fig. 3. Flow of a Newtonian fluid about a sphere.

Validity of Assumptions

The correlation of Figure 7 confirms the basic assumption made [Equation (2)] regarding the relationship between shear rate and impeller speed, at least over the range of variables investigated and in both the laminar and transition regions. The conclusion that the average shear rate, hence apparent viscosity, of the non-Newtonian fluid depends only on the rotational speed of the impeller may not, at first glance, appear to be obvious. However, supporting evidence may also be obtained from Equation (10) for the shear rate at the bob of a viscometer in an infinite fluid. In this case the rate of shear also depends only on the rotational speed, and not on dimensions of the bob. In the laminar region the analogy between a viscometer bob and the impeller of a mixer is rather close, since as a matter of fact, many commercial viscometers employ bobs which are geometrically similar to mixing impellers. In both the laminar and transition regions investigated in this work the impellers generally behaved as though in an infinite fluid since changing the distance between the tip of the impeller and the wall of the tank (i.e., changing the T/D ratio) had no effect on power consumption. As will be discussed later, this is in agreement with extensive work on Newtonian fluids. However, as the T/D ratio decreases toward unity, i.e., as the diameter of the impeller approaches that of the tank, a point must eventually be reached where the shear rate, hence power consumption, is at least partly dependent on either the T/D ratio (clearance between impeller and tank wall) or the peripheral speed of the impeller or both. It appears plausible that at T/D ratios close to unity the shear rate at the impeller may take a mathematical form similar to that for a bob-and-cup viscometer, Equation (6).

Recorrelation of data in previous papers on non-Newtonian-fluid agitation might appear desirable to extend the results of this work; however, sufficiently detailed tabulations of experimental data were not available from one source (*2*) and the other two papers (*9*, *23*) either did not measure the rheological properties of the fluid or worked with only slightly non-Newtonian systems. One may, however, cite the work of Magnusson (*9*) to extend the generality of the unique relationship between average shear rate and rotational speed of the impeller, as follows.

Equation (2) implies that scale-up of a model should be carried out at constant speed for a given non-Newtonian fluid, so that the flow properties of the fluid may be identical in the prototype and the model. Figure 7 proves this for the flat-bladed turbines of the present work, and Magnusson's data confirm this for two S-shaped paddles with diameters of 2.8 and 6.2 in. It may be concluded that since the same type of relationship between shear rate and impeller speed holds for two greatly different types of impellers, as well as for various different fluids, it appears to be of general utility. No data are available, however, to confirm the constancy of k at a value of 13.0 for systems other than those discussed here. In particular, k might be expected to vary with the flow-behavior index n of the fluid, which was not varied widely in the present studies. [Cf. Equation (10).] It should be noted in passing that scale-up procedures cannot be carried out in the usual sense of the term, however, because the Reynolds numbers will not be identical in the prototype and model when N, ρ, and μ_a are all held constant.

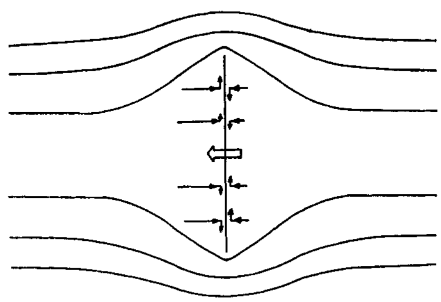

Fig. 4. Flow of a Newtonian fluid about a flat plate.

Ranges of Variables

The power range encompassed in this study was higher than the range which is used industrially for Newtonian fluids (*16*), because pseudoplastic non-Newtonian fluids are inherently more difficult to agitate. Since the exact power requirements for different levels of mixing rate are not yet known for these materials, the power input was varied over an extremely wide range to ensure applicability of the results to industrial conditions.

While the range of Reynolds numbers covered was only 130-fold, it is the range in which viscous, highly non-Newtonian fluids are most likely to be agitated. With less viscous non-Newtonians the range

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*Tabular material has been deposited as document 5119 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or 35-mm. microfilm.

may be extended to higher Reynolds numbers, but usually the same highly non-Newtonian character is not observed in such less viscous systems.

Effect of Geometric Variables

The position of the impeller did not critically affect power consumption when vortices were not formed. Vortices could be formed, however, if the impeller were placed closer to the surface than specified in Appendix B.

No consistent effects due to baffling or T/D ratios greater than 2 were found, which is in agreement with the work of Rushton et al. (*22*). Hirsekorn and Miller (*5*) report that in the laminar region T/D can be made much smaller than the value of 2 reported by Rushton. Runs 13 and 17, with T/D ratios of 1.33 and 1.45 respectively, indicate that these conclusions are valid also for non-Newtonian materials in both the laminar and transition regions. However, the number of data points available is too small to support such a conclusion firmly.

Similarity to Pipe Flow

An interesting similarity appears to exist between the means of correlating pipe-flow data and mixing-power–consumption data which suggests an alternate means of correlation for the latter. For a non-Newtonian fluid which obeys Equation (1) it may be readily shown that the friction factor for pipe flow is a function of $(D^nV^{2-n})/g_cK \cdot 8(n/6n + 2)^n$, which is a special form of the generalized number $(D^{n'}V^{2-n'}\rho)/\gamma$ proposed for pipe-flow work (*12*). One will notice that these groups both degenerate to the usual Reynolds number for Newtonian fluids ($K = \mu/g_c$ and $n = 1.00$). Application of dimensional analysis to the mixing problem (with the only important length dimension assumed to be the impeller diameter) shows that the power number must be a function of $(D^2N^{2-n}\rho)/g_cK$ and n. The function of n cannot be evaluated for mixing work at present but the assumption that it is the same as in the pipe-flow case, i.e. power number is a function of $[(D^2N^{2-n}\rho)/g_cK][8(n/6n+2)^n]$ or $(D^2N^{2-n}\rho)/\gamma$, yields a Reynolds number which is numerically proportional to the $(D^2N\rho)/\mu_a$ reported here. Further, more extensive data are needed to determine whether use of the generalized Reynolds number in mixing work is of widespread generality or merely a coincidence in the case of the present data.

Quality of Mixing

The type of flow or the quality of mixing is, of course, the end result of the choice of a given agitator. However, as mentioned before, the logical procedure for studying agitation is first to determine the amount of power dissipation and then to turn to the more complex problem of mixing patterns and rates. The measures of the rates of mixing studied to date (Table 4) are, therefore, qualitative. Nevertheless, several important conclusions may be drawn at this time concerning the comparative rates of mixing of Newtonian and non-Newtonian fluids at a given power input per unit volume of fluid.

Hirsekorn and Miller (*5*) found that particles could be suspended in viscous Newtonian fluids during agitation within the laminar region (N_{Re} below 10). The maximum power input required to suspend the particles was 6.0 hp./1,000 gal. of fluid. While the data of Table 4 are very irregular in the sense that Reynolds numbers and power requirements for movement of all the fluid vary widely, in only one case out of thirteen was reasonably complete fluid movement attained in the laminar region and, again in only one case, was any fluid movement at the tank wall noticeable at power inputs as low as 6.0 hp./1,000 gal. Although decreasing the T/D ratio to nearly unity would almost certainly reduce the Reynolds numbers, and hence the power required for complete movement of the fluid, it appears questionable whether this power reduction would be great enough to bring the power consumption at a given mixing rate down to the level of Newtonian fluids. Most of the data of Table 4 do, however, show an appreciable decrease in power required to mix a given fluid completely (as defined by movement at the tank wall) as T/D is decreased. Such complete fluid turnover does not, on the other hand, imply higher shear rates, as these are probably affected adversely by use of larger impellers (lower T/D ratios) and lower rotational speeds, as shown by Equation (2). Thus it is more difficult to obtain simultaneously both high rates of fluid turnover and high shear rates in these non-Newtonian systems without high power requirements.

Until really definitive data become available, adoption of the median values of power input and Reynolds numbers given in Table 4 is suggested as the criterion below which mixing will frequently be ineffective in these non-Newtonian systems. Some confirmation of these high power-input levels may be obtained from industrial practice: in one installation the mixing of non-Newtonian fluids is in the range of 5 to 50 hp./1,000 gal. in full-scale equipment, while pilot plant work (indicative of future operations) ranges from 50 to 160 hp./1,000 gal. This industrial experience also indicates that to date no enormously improved agitator or system designs which might be particularly suitable for the non-Newtonian fluids described here have been developed. Fundamentally, this problem arises because the apparent viscosity of a pseudoplastic fluid increases with distance from the impeller; hence the fluid tends to "set up," or

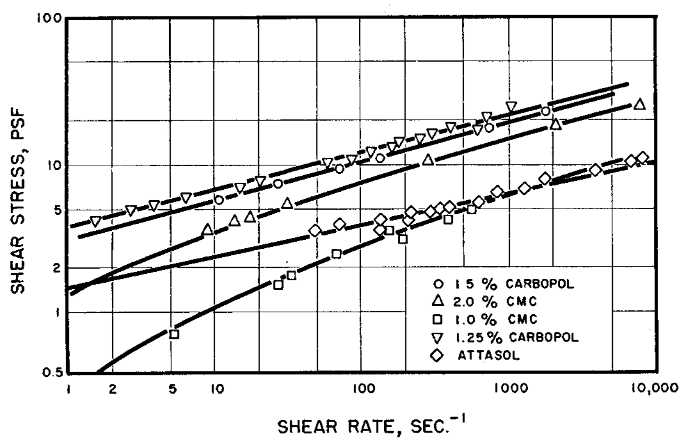

Fig. 5. Flow curves of fluids used.

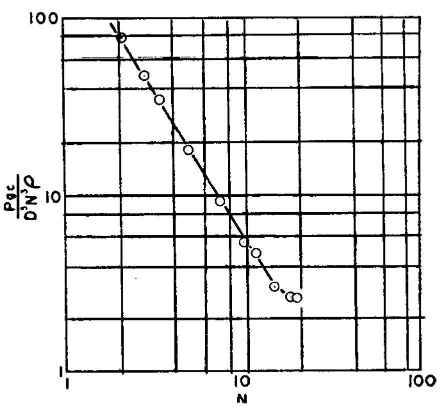

Fig. 6. Power number vs. impeller speed for 2.5% CMC.

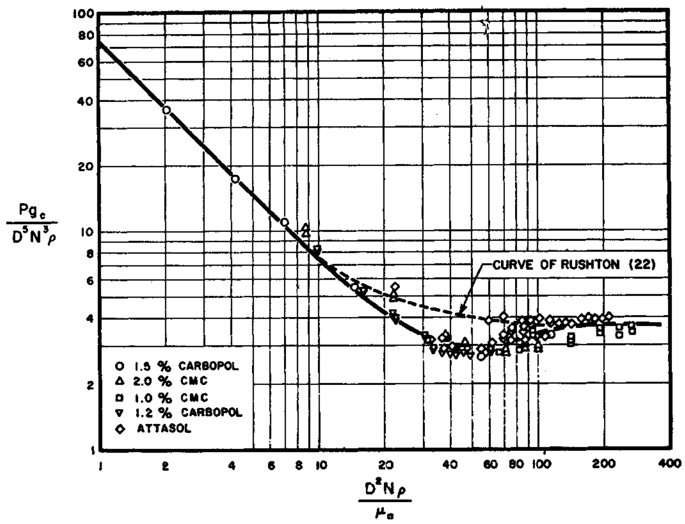

Fig. 7. Power-number–Reynolds-number curve for non-Newtonian fluids; all points in the crowded regions not shown.

as well as non-Newtonian fluids requires quantitative study.

4. Extension of the present work to a greater variety of non-Newtonian fluids, including dilatant materials, should be of considerable interest to prove conclusively whether or not the value of k [Equation (2)] depends on the flow behavior index of the fluid and whether the generalized Reynolds number is truly applicable to correlation of power-consumption data for all fluids.

ACKNOWLEDGMENT

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remain motionless, under conditions where a Newtonian fluid is mixed relatively completely. Therefore the problem is somewhat alleviated by use of multiple impellers inside a single tank and by use of very low T/D ratios in the industrial examples cited.

Design Procedure

Within the laminar region ($N_{Re} < 20$ when n ranges between about 0.25 and 0.45, and $N_{Re} < 10$ when $n = 1.00$) conditions are sufficiently well defined for the mixing equation, which is somewhat analogous to Poiseuille's law for friction in a round tube, to be written as suggested by Rushton and Oldshue (*21*). This equation is the same for Newtonian and non-Newtonian fluids:

$$N_p = 71/N_{Re} \qquad (3)$$

Substitution of the definitions of the Reynolds numbers used in this work gives

$$P = \frac{71\mu_a}{g_c} D^3N^2 \qquad (4)$$

It has been suggested that these equations are particularly useful for design purposes. The constant given was developed for Mixco turbines with six flat blades and is dependent on the particular type of impeller used. As would be expected from the analogous problem of flow in tubes (*12*), highly non-Newtonian fluids with a flow-behavior index n near zero show a smaller change of power with impeller speed than do Newtonian fluids ($n = 1.00$), since μ_a decreases as N increases.

Under any flow conditions the recommended procedure for estimating power consumption once the type, size, and speed of the impeller have all been fixed may be reviewed as follows:

1. Knowing N, one may evaluate kN [Equation (2)] to determine the average shear rate in the system.
2. From viscometric data (shear stress vs. shear rate) for the fluid in question, μ_a is calculated at the above-average shear rate.
3. The Reynolds number $(D^2N\rho)/\mu_a$ is then calculated and the corresponding power number read from Figure 7.

As the procedure is empirical, extrapolation of the variables beyond the ranges covered cannot be recommended.

CONCLUSIONS AND SUMMARY

1. The assumption that average fluid shear rates are related only to impeller speed has led to an understanding and correlation of the power requirements for agitation of non-Newtonian fluids. The quantitative relationships, which are applicable to both Newtonian and non-Newtonian fluids over the ranges of variables investigated, represent a simple generalization of the well-known results from Newtonian systems.

2. The laminar region may extend to higher Reynolds numbers in pseudoplastic fluids than in Newtonian systems.

3. Preliminary qualitative observations indicate that more power is required for the rapid mixing of highly non-Newtonian systems than for Newtonian fluids.

RECOMMENDATIONS FOR FURTHER WORK

1. Although data were taken on 8-in. impellers, the limitations of the present equipment did not allow high rotational speeds for these larger systems. Therefore, in order to approach plant-scale conditions more closely one must extend the data to larger and more powerful mixing systems.

2. In some commercial installations the T/D ratio may be smaller than the values studied in the present work. Limited data indicate that the power consumption may not be significantly affected by such changes, but this influence of small values of T/D should be investigated further for both Newtonian and non-Newtonian fluids.

3. The quality of mixing of Newtonian

NOTATION

NOTE:—Since the final correlation is based on dimensionless groups, any consistent set of units may be used. The units given in the following table refer to those used by the authors in the data tables unless specific units are given.

B = width of baffles, ft.
C = height of impeller off bottom of tank, ft.
D = impeller diameter or bob diameter, ft.
g = gravitational acceleration, ft./sec.^{2}
g_c = conversion factor, (lb. mass)(ft.)/(lb. force)(sec.^{2})
h = height of viscometer bob, ft.
K = fluid property in Equation (1), (lb. force)(sec.^{n}/sq. ft.)
k = proportionality constant, dimensionless
L = length of pipe or capillary, ft.
N = rotational speed, rev./sec.
N_p = power number, dimensionless, $Pg_c/D^5N^3\rho$
N_{Re} = Reynolds number, dimensionless, taken as $(D^2N\rho)/\mu_a$
n = flow-behavior index, dimensionless

$$n' = \frac{d \ln (\Delta pD/4L)}{d \ln (8Q/\pi D^3)}$$

$$n'' = \frac{d \ln (2t/\pi D_i^2h)}{d \ln (4\pi N/1 - 1/s^2)}$$

P = power, (ft.)(lb. force)/sec.
p = pressure, lb. force/sq. ft.
Q = flow rate, cu. ft./sec.
r = radius, ft.
S = scale reading, lb. force
s = D_o/D_i
T = tank diameter, ft.
t = torque, ft.(lb. force)
u = point velocity, ft./sec.
V = volumetric average velocity, ft./sec.

γ = generalized viscosity coefficient, $\gamma = (g_c K)/8 \cdot (6n + 2/n)^n$, lb. mass/(ft.)(sec.$^{2-n'}$)
Δ = difference of
μ_a = apparent viscosity, lb. mass/(ft.)(sec.)
ρ = density, lb. mass/cu. ft.
τ = shear stress, lb. force/sq. ft.

Subscripts

i = bob wall
0 = cup wall

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APPENDIX

A. Definitions of Non-Newtonian Behavior

Figure 8 illustrates on arithmetic coordinates the classical definitions used by rheologists. These definitions are based on the relationship between the shearing stress imposed on a fluid, τ, and the resulting shear rate, du/dr, as measured in various kinds of viscometers. Newtonian fluids show the familiar straight-line relationship, the slope of the line being defined as the *viscosity* of the fluid. A Bingham-plastic fluid is defined as one which also shows a linear relationship between shear stress and shear rate, but the relationship does not pass through the origin. Pseudoplastic and dilatant fluids do not show a linear relationship. Figure 9 shows the equivalent logarithmic plot.

For the Newtonian fluid the ratio of shear stress divided by shear rate is a constant and defined as the viscosity of the fluid. For the non-Newtonian, the equivalent, or "apparent," viscosity is not a constant. Instead the ratio of shear stress divided by shear rate changes with shear rate. For Bingham-plastic and pseudoplastic materials this ratio decreases with increasing values of shear rate; for dilatant materials it increases.

There are two other kinds of non-Newtonian behavior, termed *thixotropy* and *rheopexy*. The apparent viscosities of thixotropic and rheopectic fluids depend on the time of shear as well as on rate of shear. These two kinds of behavior have been too complex to study and, since they are of less frequent industrial importance, will not be discussed here. Industrially, pseudoplastic behavior is probably more important than the other types of non-Newtonian behavior combined.

All the foregoing definitions are restricted to materials which do not exhibit elastic recovery or "viscoelasticity." That is to say, once they have been sheared there is no tendency for the fluid to return to its original shape or configuration. The necessity of this assumption may prove to be a more serious limitation of the present work than the assumed absence of thixotropy and rheopexy but cannot be dealt with until the engineering problems of design for pseudoplastic, Bingham-plastic, and dilatant behavior have been well-developed.

These classifications are discussed in more detail elsewhere (*1, 4, 11, 13, 19*, for example). Many engineering publications have been concerned with fluids which were believed to be of the Bingham-plastic type. At the beginning of the present experimental program extensive determinations of the shear-stress–shear-rate relationships of fluids which have been claimed to be Bingham plastics led to the conclusion that true Bingham plastics probably exist only very rarely, if at all. Except for a few specially prepared materials this type of behavior broke down over shear-rate ranges greater than about 1:100.

B. Experimental Apparatus and Procedure

The mixing equipment used in this work is shown in Figures 10 and 11. It consisted of a ⅓-hp. variable-speed motor, four Mixco Standard flat-bladed turbines (Mixing Equipment Co.), and four cylindrical tanks. To measure torque accurately, the motor was mounted on a large ball-bearing ring fixed in a cast-aluminum plate; the motor rotated freely with very little friction. The reaction torque developed by the motor in driving the turbine was taken from the motor by a torque ring attached to the motor head and transferred by a small, essentially frictionless pulley to a dynamometer scale. The turbine shaft was fitted inside a special hollow shaft provided on the motor by the manufacturer (Mixco) so that the shaft height was variable. Turbine speeds were measured by an electric tachometer geared to the motor shaft.

Four cylindrical flat-bottomed tanks 6, 8.2, 11.6, and 22 in. in diameter were used in this work. The smallest was a beaker, the next two were Pyrex tanks, and the final one was a 55-gal. drum. All tanks except the smallest one were fitted with four removable baffles with a width of one tenth of the tank diameter.

The impellers which were used had diameters of 2, 4, 6, and 8 in. The ratios of the width and length of the impeller blades relative to their diameter were 1:5 and 1:4 respectively. With the smallest flat-bladed turbine the torque readings were too low

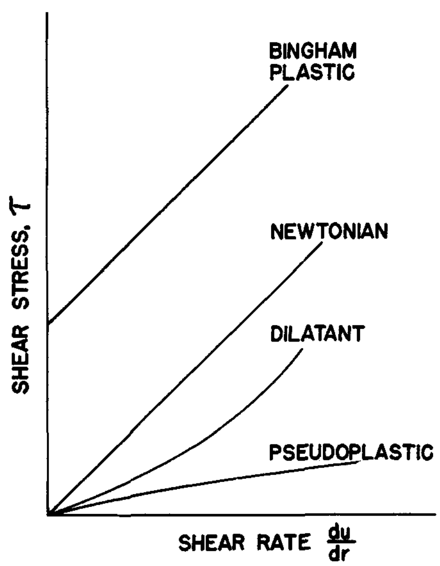

Fig. 8. Fluid characteristics (arithmetic scale).

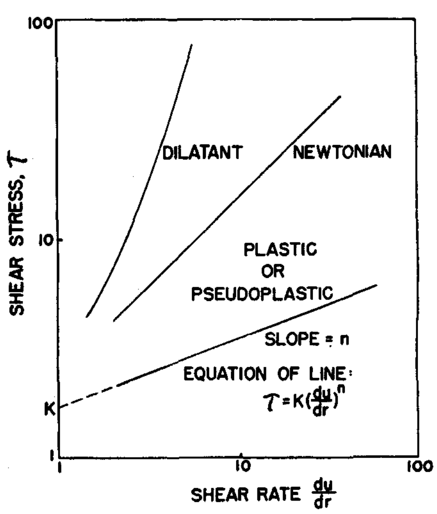

Fig. 9. Fluid characteristics (logarithmic scale).

for the dynamometer scale; hence a torque table was improvised from a dead-weight tester connected through a system of pulleys to a smaller spring scale.

In order not to vary a large number of geometric variables at once, the fluid level in the tanks was maintained at one tank diameter. The impeller was placed at one impeller diameter off the bottom of the tank unless this placement brought the impeller within one impeller diameter of the fluid surface. In this case the impeller was centered halfway between the liquid surface and the tank bottom.

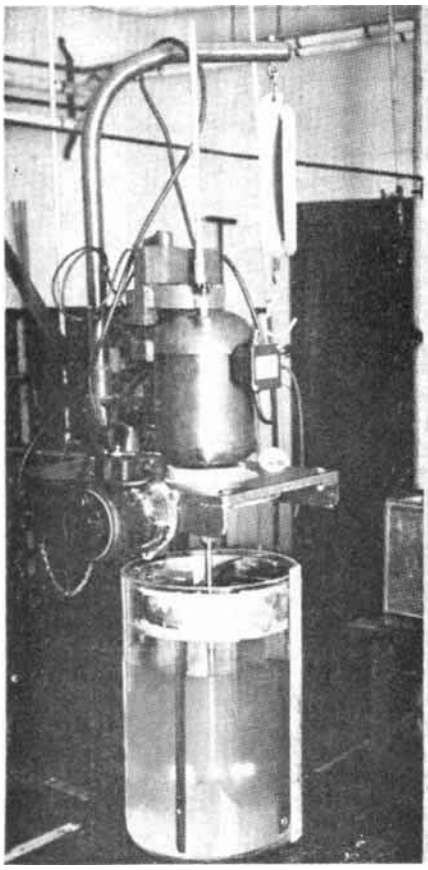

Fig. 10. Mixing equipment.

Rheological properties were measured both on a Stormer rotational viscometer and with a capillary-tube viscometer (Figure 11). The capillary-tube viscometer was an instrument used for the extrusion of plastics (*24*).

The fluid to be tested was placed in a stainless steel pressure chamber and extruded under pressure from a capillary tube mounted in the bottom of the chamber. The flow rate of the fluid was measured by use of a stop watch and a weighing balance. The extrusion pressure was supplied by a nitrogen cylinder and was measured on laboratory test gauges. During the experimental runs the pressure chamber was surrounded by water to control the temperature.

The Stormer viscometer was modified in that a smooth-walled cylinder was used for the cup. End effects on the bob were corrected by calculation of the equivalent bob height from data taken with National Bureau of Standards calibrated oils. This equivalent bob height was about 25% greater than the actual height. The fact that the data (Figure 5 and Table 2) from both instruments coincide within experimental error supports the adequacy of this procedure for accounting for end effects in the rotational viscometer as well as for the absence of any similar problems with the capillary-tube viscometer.

Detailed equipment dimensions are given in Table 1.

C. Viscometry

General equations (which do not require the presumption of a shear-stress–shear-rate relationship) have been reported in the literature; these were used exclusively to interpret the viscometric measurements.

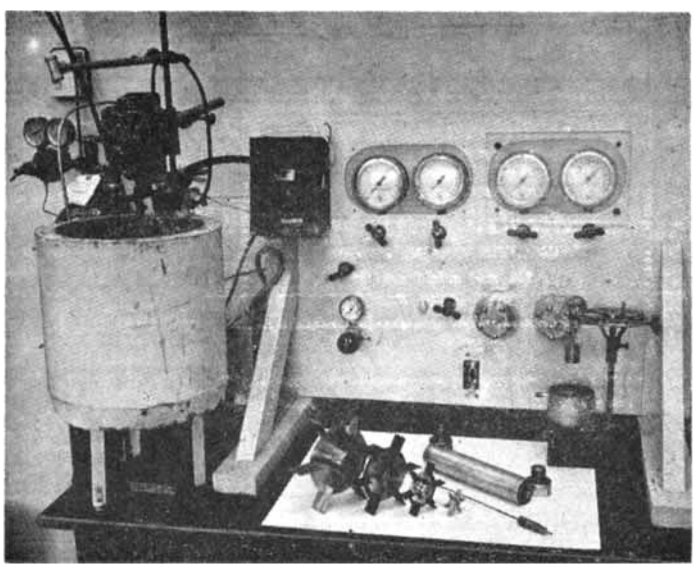

Fig. 11. Impellers and viscometric equipment.

For the rotational viscometer, Krieger and Maron (*7*) have succeeded in writing an infinite series which converges rapidly for cup-to-bob diameter ratios of less than 1.2:1. Their equations are

Shear stress at the bob:

$$\tau_i = \frac{2}{\pi}\frac{t}{D_i^2 h} \tag{5}$$

Shear rate at the bob:

$$-\frac{du}{dr} = \frac{4\pi N}{1 - 1/s^2}\left[1 + k_1\left(\frac{1}{n''} - 1\right) + k_2\left(\frac{1}{n''} - 1\right)^2\right] \tag{6}$$

n'' is evaluated at the shear stress calculated by means of Equation (5). The instrument constants k_1 and k_2 are

$$k_1 = \frac{s^2 - 1}{2s^2}\left(1 + \frac{2}{3}\ln s\right);$$

$$k_2 = \frac{s^2 - 1}{6s^2}\ln s \tag{7}$$

In any one run n'' is the slope of a logarithmic plot of the torque plotted against rotational speed, as all other factors in n'' are constant.

Rabinowitsch's solution of the motion of fluids flowing through a pipe or capillary tube gives upon rearrangement (*18*)

$$\tau = \frac{\Delta p D}{4L} \tag{8}$$

$$-\frac{du}{dr} = \frac{32Q}{\pi D^3}\left[\frac{3n' + 1}{4n'}\right] \tag{9}$$

Again n' must be evaluated at the corresponding shear stress given in Equation (8). For a given capillary tube n' is found from a logarithmic plot of pressure drop vs. flow rate.

Another general equation given by Krieger and Maron (*7*) is of use in viscometric equipment approximating a long cylinder rotating in an infinite fluid:

$$\frac{du}{dr} = \frac{4\pi N}{n''} \tag{10}$$

Brookfield viscometers (Brookfield Engineering Laboratories) may fall into this category, depending on the shapes of the bobs used. The shear stress is given by Equation (5). One cannot use the Brookfield conversion factors to determine the apparent viscosity directly because the formulas supplied with the instrument are for Newtonian fluids ($n'' = 1.00$). However, since a Brookfield has several speeds n'' can be obtained from a logarithmic plot of scale reading vs. rotational speed, and the shear rates calculated by means of Equation (10).

For a power-function non-Newtonian [Equation (1)]:

$$n = n' = n''. \tag{11}$$

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